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A Different Theory of Anomalous Magnetic Moment

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Abstract

The invariance of Dirac's equation under rotation has been used to obtain the wave equation for a particle interacting with an electromagnetic field. The origin of the anomalous magnetic moment of a particle has been attributed to the existence of mass due to spin. These masses for a few representative particles have been calculated. In particular these calculations give a mass of 592.074 eV for a neutrino. An operator for the spin angular velocity has been constructed and the values of spin angular velocities for the particles have also been calculated.

1. *Introduction*

Today there is an extensive amount of research being carried on to explain the anomalous magnetic moment of various elementary particles, and, as a result, vast literature exists on this subject. In the present paper the author has deviated from the traditional path. An exact treatment of anomalous magnetic moment based on a more realistic picture of the actual mechanism behind this phenomenon has been presented. The formalism developed in this paper gives us valuable additional information not obtainable by traditional means. In his quest, the author has not been bound by any of the past rigidly established rules. Starting with the invariance of the Dirac equation for a free particle under rotation, a wave equation in the coordinate system spinning with respect to the original one has been obtained. This equation has been used to obtain the wave equation describing a spin-$\frac{1}{2}$ particle interacting with an electromagnetic field. This equation correctly and exactly gives the intrinsic magnetic moment of a particle. The anomalous magnetic moment has been attributed to the existence of mass due to spin, which forms a part of the observed mass of the particle. The mass due to spin has been calculated for a few representative elementary particles including a neutrino. The angular velocities of spin have also been calculated for the particles. For this purpose, an operator for spin angular velocity has been constructed.

2. *The Wave Equation for a Spin-$\frac{1}{2}$ Particle in an Electromagnetic Field*

The invariance of the Dirac equation for a free particle under rotation leads to the following wave equation in a coordinate system spinning with an angular velocity $-\boldsymbol{\omega}$ with respect to the original coordinate system[1]:

$$i\hbar \frac{\partial \psi}{\partial t} = (H_0 + \boldsymbol{\omega} \cdot \mathbf{J})\psi \tag{2.1}$$

where $H_0 = -ic\hbar\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + \beta mc^2$ is the Dirac operator expression for energy of a free particle and $\mathbf{J} = \mathbf{S} + \mathbf{L}$ is the total angular momentum operator for a Dirac particle. Equation (2.1) is, therefore, to be interpreted as the wave equation for a spin-$\frac{1}{2}$ particle spinning or rotating with an angular velocity $\boldsymbol{\omega}$.

As a special case, it is known from classical electrodynamics that the gyration or the precession angular velocity of a charged particle carrying a charge e in a magnetic field $\mathbf{B}$ is

$$\boldsymbol{\omega} = -e\mathbf{B}/mc \tag{2.2}$$

Therefore, for this particular case, equation (2.1) would assume the form[2]

$$i\hbar \frac{\partial \psi}{\partial t} = \left(H_0 - \frac{e\mathbf{L}}{mc} \cdot \mathbf{B} - \frac{e\mathbf{S}}{mc} \cdot \mathbf{B}\right)\psi \tag{2.3}$$

From equation (2.3), it is clearly seen that the intrinsic magnetic moment ($\boldsymbol{\mu}_s$) due to spin is

$$\boldsymbol{\mu}_s = e\mathbf{S}/mc \tag{2.4}$$

and the magnetic moment ($\boldsymbol{\mu}_L$) due to the orbital angular momentum is

$$\boldsymbol{\mu}_L = e\mathbf{L}/mc \tag{2.5}$$

Thus we see that for $\boldsymbol{\mu}_s$ the g factor is correctly given by equation (2.4).[3] However, the value of $\boldsymbol{\mu}_L$ given by equation (2.5) is twice as large as that predicted by classical electrodynamics. It seems that since $\mathbf{S}$ and $\mathbf{L}$ both represent angular momenta, $\boldsymbol{\mu}_s$ and $\boldsymbol{\mu}_L$ should be given by the same rule and therefore equation (2.5) should be the correct equation. However, by the same argument, since the spin angular momentum can take half-integral values, the orbital angular momentum should also be allowed to assume these values. Irrespective

[1] It should be mentioned here that the same equation was obtained by Inglis (for example, see Inglis, 1954) but within a different context.

[2] It may be mentioned that if, in addition to the magnetic field, the electric field is also present, then the corresponding wave equation should be (ϕ = electrostatic potential)

$$i\hbar \frac{\partial \psi}{\partial t} = \left(H_0 + e\phi - \frac{e\mathbf{L}}{mc} \cdot \mathbf{B} - \frac{e\mathbf{S}}{mc} \cdot \mathbf{B}\right)\psi$$

Here, of course, terms like the spin-orbit interaction, etc. have not been included, but this will be the subject of future communications.

[3] We shall see in Section 3 that equation (2.4) gives the total intrinsic magnetic moment correctly including the anomalous part of the magnetic moment.

of the value for orbital angular momentum, the sign of the wave function changes as the coordinate system is rotated through an angle of 2π radians as a result of its spin. Therefore, there is no justification for restricting the eigenvalues of orbital angular momentum only to integral numbers.

3. *The Anomalous Magnetic Moment*

Before we go into the investigation of the anomalous magnetic moment, the question to be answered is as to the origin of the spin angular momentum. The only way that a particle can possess spin angular momentum is if it is actually spinning with certain angular velocity. This angular velocity, in analogy with Newtonian mechanics, must be proportional and parallel to the spin angular momentum. Thus the spin angular velocity operator $\boldsymbol{\omega}_s$ should be given by

$$\boldsymbol{\omega}_s = \lambda e\mathbf{S}/mc \tag{3.1}$$

where λ is a constant. The constant $(\lambda e/mc)^{-1}$ may be called the moment of inertia of the particle. The justification for the factor e/m on the right-hand side of equation (3.1) comes from the observation that in equation (2.2) the angular velocity is proportional to e/m. The energy assocated with the spin of the particle must form a part of the observed mass of the particle. This energy due to spin is given by the operator

$$m_s c^2 = \boldsymbol{\omega}_s \cdot \mathbf{S} = \lambda e S^2/mc \tag{3.2}$$

Consider the case wherein a charged particle enters a region of pure magnetic field perpendicular to the direction of its velocity. Then the particle will move along a circle with angular velocity as given by equation (2.2), where m would be equal to the observed mass of the particle. If the particle were spinless and therefore had no mass due to spin, then in addition to its orbital motion it would also rotate about its own body axis with exactly the same angular velocity. However, if the particle possesses spin, then the angular velocity of rotation about its own body axis would be given by equation (2.2) with m standing for the bare mass of the particle, which is the mass of the particle excluding the mass due to its spin. From now on let m represent the bare mass of the particle and m_0 its observed mass. Then it should be concluded that $\boldsymbol{\mu}_s$ and $\boldsymbol{\mu}_L$ should correctly be given by

$$\boldsymbol{\mu}_s = e\mathbf{S}/mc \tag{3.3}$$

and

$$\boldsymbol{\mu}_L = e\mathbf{L}/m_0 c \tag{3.4}$$

From the above argument, it is clear that m in equation (3.1) and therefore in equation (3.2) represents the bare mass.

Experimentally it is known that the intrinsic magnetic moment of certain particles is given by

$$\mu_s = \frac{e\hbar(1+\kappa)}{2m_0 c} \tag{3.5}$$

where κ, a constant, is characteristic of the particle. Comparing (3.3) with (3.5), we get

$$\kappa = m_0/m - 1 \tag{3.6}$$

Now let us consider a few special cases:

(a) *Electron.* In the case of electron, it is known experimentally that $\kappa_e = 0.00116$ and $m_0 = 0.511$ MeV. Substituting these values in (3.6), we get

$$m_{se} = m_{0e} - m_e = 592.074 \text{ eV} \tag{3.7}$$

Thus we see that the mass of the electron due to its spin is 592.074 eV. Using this value for m_{se}, we obtain from equation (3.1) and (3.2) the value for $\boldsymbol{\omega}_s = (\boldsymbol{\omega}_s \cdot \boldsymbol{\omega}_s)^{1/2} = 16.52 \times 10^{16}$ rev/sec. which means that the electron is spinning about its own body axis with an angular velocity of 16.52×10^{16} rev/sec.

(b) *Spin-½ Particle of Bare Mass Zero.* Next let us consider a particle that is similar to an electron in every respect except for its bare mass, which we take to be vanishingly small. Let us assume that the ratio e/m is the same as that for an electron, which therefore means that the charge of this particle is also vanishingly small. Then from equation (3.3) we find that its intrinsic magnetic moment is the same as that for an electron. If we further assume that the value of λ for this particle is also the same as that for the electron, then the observed mass of such a particle, which obviously is due only to its spin, should be the same as the mass of the electron due to its spin, i.e., 592.074 eV, and it should be spinning about its own body axis with the same angular velocity as the electron, i.e., 16.52×10^{16} rev/sec. Perhaps this particle can be identified with a neutrino.

An analysis similar to that for the electron can easily be carried on for other "particles." For example, for the muon we get $m_{s\mu} = 158.249$ keV and $\boldsymbol{\omega}_{s\mu} = 44.15 \times 10^{18}$ rev/sec, and for the proton we get $m_{sp} = 603.1$ MeV (assuming that besides its bare mass, the rest of its mass is all due to spin) and $\omega_{sp} = 16.98 \times 10^{22}$ rev/sec etc.

Reference

Inglis, D. R. (1954). *Physical Review*, **96**, 1059.